Homework 11

1. Find the indefinite integral using the substitution $x = 4\sin\theta$.

$$\int \frac{1}{(16 - x^2)^{\frac{3}{2}}} dx$$

2. Find the integral.

$$\int \cos^5 x \sin x \, dx$$

3. Use partial fractions to find the integral.

$$\int \frac{5 - x}{2x^2 + x - 1} \mathrm{d}x$$

Sol：

1.

$$dx = 4\cos\theta d\,\theta, \sqrt{16-x^2} = 4\cos\theta$$

$$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4\cos\theta}{(4\cos\theta)^3} d\theta = \frac{1}{16}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{16}\tan\theta + c = \frac{1}{16}\frac{x}{\sqrt{16-x^2}} + c$$

2.

Let $u = \cos x\,, du = -\sin x\, dx$

$$\int \cos^5 x \sin x\, dx = -\int u^5\, du = -\frac{u^6}{6} + C = -\frac{\cos^6 x}{6} + c$$

3.

$$5 - x = A(x+1) + B(2x-1)$$

When $x = \frac{1}{2} \to A = 3\,, x = -1 \to B = -2$

$$\int \frac{5-x}{2x^2+x-1} dx = 3\int \frac{1}{2x-1} dx - 2\int \frac{1}{x+1} dx$$

$$= \frac{3}{2}\ln|2x-1| - 2\ln|x+1| + c$$